NONSTATIONARY FLOW AROUND DELTA WINGS BY A STREAM BEHIND SHOCKS

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The incidence of a shock wave on a moving wing results in the origination of a complex spatial nonstationary flow and a change in its aerodynamic characteristics. The nonstationary problem of shock interaction with a delta wing at supersonic flight speeds was solved numerically earlier within the framework of the linearized theory [1] and in a complete nonlinear formulation [2]. The impulses of the nonstationary forces and moments during the whole interaction time were evaluated in [3].

This paper is devoted to an analytic determination of the nonstationary aerodynamic characteristics of delta wings under the incidence of shocks in a linearized formulation by using the analogy with the problem of entry into an equivalent vertical gust. The results obtained can be used to perform rapid and sufficiently exact estimates (conversions) of instantaneous aerodynamic force and moment values for an arbitrary law of pressure variation behind the wave front.

1. Let us examine the problem of weak shock incidence on a plane delta wing flying at a supersonic velocity V without slip at a zero angle of attack. The assumptions made permit utilization of the linearized theory of potential flows. We introduce a rectangular Oxyz coordinate system connected to the wing, whose origin is superposed on the wing apex, the Ox axis is directed oppositely to the flight velocity vector, the Oy axis is perpendicular to the plane of the wing, and the Oz axis is along the span.

Let us first find the solution for the case when the excess pressure behind the wave front is constant (Δp = const, $\Delta p/p \ll 1$) and a uniform co-flow with velocity $V_1 = (\Delta p/p)a/\varkappa$ moves behind it (p and a are the pressure and sound speed in the gas at rest, and $\varkappa$ is the adiabatic index). The influence of the shock is that a small downwash angle $\alpha = v/V$ is created on the part of the wing it encloses because of the vertical component v of the co-flow velocity. We call that gust, which is displaced in space and produces the very same downwash distribution as does the incident shock on the wing, equivalent vertical. Then the analogy to the problem of wing entry into an equivalent vertical gust can be used to solve the problem under consideration.

Let us note that during the flow around an infinite span wing (flat plate), there is the possibility of compiling a solution obtained on the basis of the analogy with the problem of entry into an equivalent vertical gust with the exact solution found by another method and thereby confirming the legitimacy of this analogy. Indeed, the two-dimensional problem of nonstationary flow by a weak shock around a plate moving at a constant supersonic speed is, as is known [4, 5], reduced within the framework of linearized theory to a three-dimensional stationary problem of the flow at a small angle of attack around a flat wing with rectangular supersonic leading and trailing edges by a uniform gas flow with Mach number $M_\infty = \sqrt{2}$. The appropriate pressure distribution is found in analytic form by using the well-developed apparatus of linear wing theory in a supersonic flow [6]. On the other hand, exactly the same pressure distribution is obtained as a result of simple calculations if the analogy between the flow around a plate by a displacing shock and its entry into an equivalent vertical gust is considered valid and the known solution of the problem of plate entry into a gust with arbitrary downwash distribution is used [7]. Therefore, the analogy mentioned yields an exact solution of the problem of the flow around an infinite span with moving at supersonic velocity in the presence of a weak shock on it.

This analogy is used in this paper in the more general case of a finite-span delta wing, as had been done in the numerical approach [1, 8]. In contrast to the solution presented in [9], the analytic solution obtained is sufficiently simple and has a totally reviewable form that makes it accessible for practical estimates. As a comparison showed, it is in conformity with known results of the theory of nonstationary flow around wings.

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2. Let the wing have supersonic leading edges (the half-angle at the apex is $\varphi_0 > \arcsin 1/M$). Then the solution of the problem of entry into a gust is known for any downwash distribution [7], but it appears most simply if the downwash is independent of the lateral coordinate z: v = v(x, t). In this case the integral of the perturbed velocity potential with respect to the span on the upper wing surface

$$\psi(x, t) = \frac{1}{Vc} \int_{-x\,\mathrm{tg}\,\varphi_0}^{x\,\mathrm{tg}\,\varphi_0} \varphi(x, y, z, t)\big|_{y=+0}\, dz \tag{2.1}$$

is represented by the quadrature [7]

$$\psi(x, t) = \frac{2\,\mathrm{tg}\,\varphi_0}{\pi\beta} \int_0^x \xi\, d\xi \int_0^\pi v\,[\xi, t - (x - \xi)\, g(\theta)]\, d\theta, \tag{2.2}$$

$$g(\theta) = \frac{\mathrm{M} + \cos\theta}{\beta}, \quad \beta = \sqrt{\mathrm{M}^2 - 1}, \quad \mathrm{M} = V/a.$$

Such a gust corresponds to incidence of a plane shock for which the normal to the front is parallel to the vertical plane of wing symmetry. Both counter- and overtaking-wave interaction with the wing is possible here. We take as beginning of the time t, the time of its arrival at the apex or at the trailing edge, respectively. Then the downwash distribution in the former case is

$$v(x, t) = \alpha H(t - k_1 x), \tag{2.3}$$

and in the latter, being realized for $M \sin\gamma < 1$,

$$v(x, t) = \alpha H[t - k_2(1 - x)], \tag{2.4}$$

where $\alpha = (V_1/V)\cos\gamma \ll 1$; $k_{1,2} = \sin\gamma/(1 \pm M\sin\gamma)$; H is the Heaviside step function, and γ is the acute angle of wave incidence at the wing. Here and henceforth all the linear dimensions are referred to the length c of the wing root chord, and the time to c/a.

Since the potential φ is an odd function of the coordinate y (antisymmetric problem) then by using a known formula from linear theory for the pressure coefficient [7], the lift coefficient for the section x = const is expressed in terms of the derivatives of the function ψ:

$$L(x, t) = \frac{4}{\mathrm{tg}\,\varphi_0}\left(\psi_x + \frac{1}{\mathrm{M}}\psi_t\right). \tag{2.5}$$

Substituting (2.2) and (2.3) here, we find in the counter-interaction case

$$L(x, t) = \frac{8\alpha}{\pi\beta} \int_0^\pi \frac{d\theta}{k_1 - g(\theta)} \int_{\eta_1(\theta)}^{\eta_2} \left[H(\eta) - \frac{1}{\mathrm{M}(1 + \mathrm{M}\sin\gamma)} \frac{\eta - t + xg(\theta)}{k_1 - g(\theta)}\, \delta(\eta)\right] d\eta,$$

$$\eta = \eta_2 - [g(\theta) - k_1](x - \xi)$$

(δ is the Dirac function, and $\eta_1(\theta) = t - xg(\theta)$, $\eta_2 = t - k_1 x$). The nonstationary lift and longitudinal moment coefficients relative to the wing apex are determined from the formulas

$$c_y(t) = \alpha C(t) = \int_0^1 L(x, t)\, dx, \quad m_z(t) = \alpha m(t) = -\int_0^1 xL(x, t)\, dx. \tag{2.6}$$

After evaluating the quadratures we obtain the following formulas for the aerodynamic coefficients (transition functions) C(t) and m(t) within different time intervals:

for $0 \le t \le k_1$

$$C(t) = \frac{4}{\mathrm{M}}\left(\frac{t}{k_1}\right)^2, \quad m(t) = -\frac{8}{3\mathrm{M}}\left(\frac{t}{k_1}\right)^3,$$

for $k_1 \le t \le t_B = 1/(M + 1)$

$$C(t) = \frac{4}{\mathrm{M}}\left(\frac{t}{k_1}\right)^2 + 4F_C(t, \mathrm{M}, \gamma), \quad m(t) = -\frac{8}{3\mathrm{M}}\left(\frac{t}{k_1}\right)^3 - \frac{4}{3}F_m(t, \mathrm{M}, \gamma),$$

for $t_B \le t \le t_C = 1/(M - 1)$

$$C(t) = \frac{4}{\pi} F_C(t, M, \gamma) \arccos \frac{t(M + \sin\gamma) - 1}{t(1 + M\sin\gamma) - \sin\gamma} + \frac{4}{\pi M}\left(\frac{t}{k_1}\right)^2 \arccos \frac{Mt - 1}{t} -$$
$$- \frac{4\sin\gamma}{\pi M k_1 \cos^2\gamma}\left(\frac{t}{k_1} - 1\right)\sqrt{t^2 - (Mt - 1)^2} + \frac{4}{\pi\beta}\arccos(M - \beta^2 t),$$

$$m(t) = -\frac{4}{3\pi} F_m(t, M, \gamma) \arccos \frac{t(M + \sin\gamma) - 1}{t(1 + M\sin\gamma) - \sin\gamma} -$$
$$- \frac{8}{3\pi M}\left(\frac{t}{k_1}\right)^3 \arccos \frac{Mt - 1}{t} - \frac{4\sqrt{t^2 - (Mt - 1)^2}}{3\pi M k_1 \cos^2\gamma} \times$$
$$\times \left\{2\sin\gamma - \frac{t}{k_1}[t(1 + \sin\gamma)(2 - \sin\gamma) + \sin\gamma]\right\} - \frac{8}{3\pi\beta}\arccos(M - \beta^2 t),$$

where
$$F_C(t, M, \gamma) = \frac{1}{M\cos^3\gamma}\left\{\left[1 + \left(\frac{t}{k_1}\right)^2\right](1 + M\sin^3\gamma) - \frac{2t}{k_1^2}(\sin^3\gamma + t\cos^2\gamma)\right\};$$

$$F_m(t, M, \gamma) = \frac{1}{M\cos^3\gamma}\left\{\left[2 + \left(\frac{t}{k_1}\right)^3\right](1 + M\sin^3\gamma) - \frac{3t}{k_1^2}\left(\sin^3\gamma + \frac{t^2}{k_1}\cos^2\gamma\right)\right\}.$$

As time t_C passes from the time of the beginning of shock interaction with the wing, the phase of the nonstationary is terminated and its stationary flow at an angle of attack α is set up for which

$$C = 4/\beta, \; m = -8/3\beta. \tag{2.7}$$

According to the formulas obtained, the nonstationary aerodynamic coefficients C and m of a wing with supersonic edges are independent of the sweepback angle as in the steady state flow case. The substantially nonmonotonic dependences C(t) and m(t) are shown in Fig. 1 ($M = 2$, $\gamma = 20, 40, 60°$ for curves 1-3) for a counterinteraction; the dashed lines are results of computations in a quasistationary approximation that yields a monotonic change in these quantities with a very much more early emergence at the steady value.

Arranging the dependence $c_y(t)$, we find that the impulse of the nonstationary part of the lift coefficient during the whole time of wave interaction with the wing $I = \int_0^{t_C} (c_y - 4\alpha/\beta)\,dt$ equals $I = -(4\alpha/3(M^2 - 1)^{3/2})[1 + (2M(M^2 - 1)\sin\gamma)/(1 + M\sin\gamma)]$. Exactly the same expression is obtained for I in [3] by the method of [10] without requiring knowledge of the instantaneous values $c_y(t)$.

In the case when the shock overtakes the wing, we have from (2.2) and (2.4)-(2.6):

in the interval $0 \le t \le k_2$

$$C(t) = 4G_C(t, M, \gamma), \; m(t) = -4G_m(t, M, \gamma),$$

for $k_2 \le t \le t'_B = t_B + k_2$

$$C(t) = 4G_C(t, M, \gamma) + \frac{4}{M}\left(\frac{t}{k_2} - 1\right)^2, \quad m(t) = -4G_m(t, M, \gamma) + \frac{8}{3M}\left(\frac{t}{k_2} - 1\right)^3,$$

for $t'_B \le t \le t'_C = t_C + k_2$

$$C(t) = \frac{4}{\pi} G_C(t, M, \gamma) \arccos \frac{(M - \sin\gamma)(t - k_2) - 1}{(1 - M\sin\gamma)t} + \frac{4}{\pi M}\left(\frac{t}{k_2} - 1\right)^2 \arccos \frac{M(t - k_2) - 1}{t - k_2} + \frac{4}{\pi\beta}\arccos[M - \beta^2(t - k_2)] +$$
$$\frac{4t\sin\gamma}{\pi M k_2^2 \cos^2\gamma}\sqrt{(t - k_2)^2 - [M(t - k_2) - 1]^2},$$

$$m(t) = -\frac{4}{\pi} G_m(t, \mathrm{M}, \gamma) \arccos \frac{(\mathrm{M} - \sin\gamma)(t - k_2) - 1}{(1 - \mathrm{M}\sin\gamma) t} +$$

$$+ \frac{8}{3\pi\mathrm{M}} \left(\frac{t}{k_2} - 1\right)^3 \arccos \frac{\mathrm{M}(t - k_2) - 1}{t - k_2} - \frac{8}{3\pi\beta} \arccos [\mathrm{M} - \beta^2 (t - k_2)] -$$

$$- \frac{4\sqrt{(t - k_2)^2 - [\mathrm{M}(t - k_2) - 1]^2}}{3\pi \mathrm{M} k_2 \cos^2\gamma} \left\{2\sin\gamma - \left(\frac{t}{k_2} - 1\right) \times \right.$$

$$\left. \left[\left(\frac{t}{k_2} - 1\right)(k_2 \cos^2\gamma + \sin\gamma) - \sin\gamma\right]\right\},$$

where

$$G_C(t, \mathrm{M}, \gamma) = \frac{1}{\mathrm{M}\cos\gamma}\left(2 - \frac{t}{k_2}\right)\frac{t}{k_2} + \frac{t^2}{\mathrm{M}k_2^3} \operatorname{tg}^3\gamma;$$

$$G_m(t, \mathrm{M}, \gamma) = \frac{2}{\mathrm{M}\cos\gamma}\left[1 - \frac{t}{k_2}\left(1 - \frac{t}{3k_2}\right)\right]\frac{t}{k_2} + \frac{t^2}{\mathrm{M}k_2^3}\left(1 - \frac{t}{3k_2}\right) \operatorname{tg}^3\gamma.$$

The overtaking interaction is characterized by a more continuous stage of nonstationary flow around the wing $t_C' = t_C + k_2$, after which the aerodynamic coefficients reach their steady-state values (2.7). The dependences C(t) and m(t) are monotonic (Fig. 2) and qualitatively similar to their quasistationary approximation (dashed lines). The graphs are constructed for M = 2 (γ = 10, 15, 20° for curves 1-3).

For perpendicular shock incidence ($\gamma = 0$) the formulas obtained above simplify and yield, in the limit as $\gamma \to 0$ after expanding the indeterminacies, expressions for the transition functions of the lift and longitudinal moment coefficients of a delta wing for a step change in the angle of attack, as should have been expected (see [8, 11], say).

The dependences C(t), m(t) presented above correspond to a constant (or very slowly varying) excess pressure behind the wave front. If its changes are substantial, then within the framework of the linearized theory utilized, they can be taken into account by using the Duhamel integral [1, 8, 11]. For instance, if the excess pressure during the dimensionless time T diminishes linearly from the initial value Δp to zero and then remains equal to zero, then the increments in the aerodynamic characteristics have the form $C_l(t) = C(t) - \frac{1}{T}\int\limits_{\substack{0\\(t-T)}}^{t} C(\tau)\,d\tau$, and analogously for $m_l(t)$. Taken here as lower limit of integration is the greater of the quantities indicated in the parentheses. Graphs of the functions $C_l(t)$, $m_l(t)$ are given in Figs. 3a and b for counterinteraction (M = 1.5, γ = 20°), and different values of the duration T of the acting pressure impulse (T = 1, 2, 5, for curves 1-3).

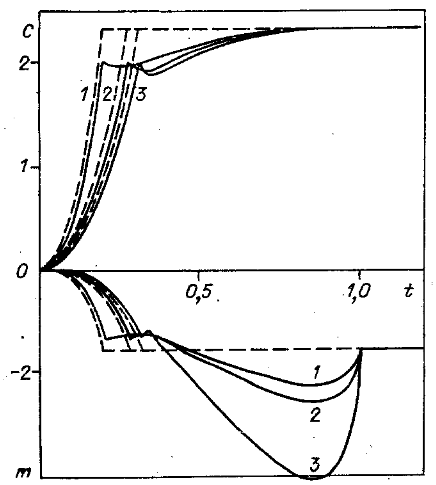

Fig. 1

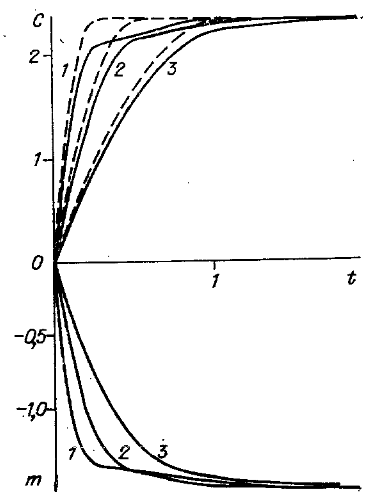

Fig. 2

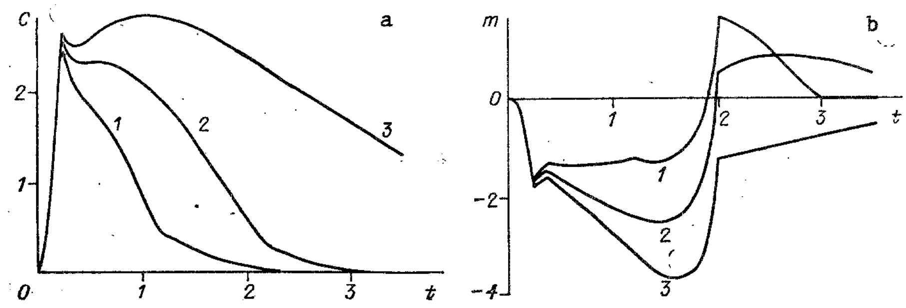

Fig. 3

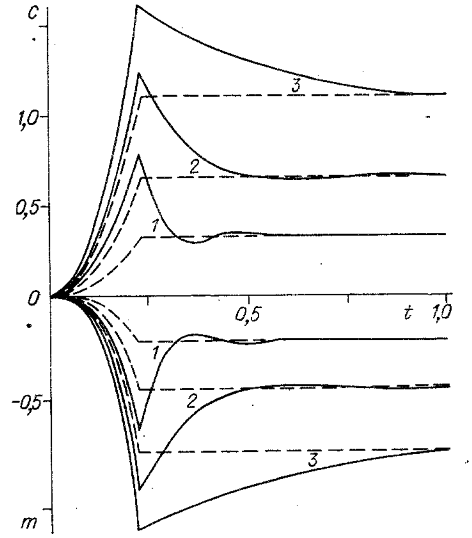

Fig. 4

3. Let us consider a greatly sweptback wing. Here the angle is $\varphi_0 \ll 1$ and the theory of nonstationary flow around a wing of ultimately small span is applicable, resulting in the law of plane sections. The shock front moving oppositely to the wing arrives at a section with coordinate x at the time $t = k_1 x$. Consequently, by using the results of [7], we write the integral of the potential with respect to the span (2.1) in the problem of wing entry into an equivalent gust with the downwash distribution (2.3) in the form

$$\psi = \alpha (x \operatorname{tg} \varphi_0)^2 \chi \left(\frac{t - k_1 x}{x \operatorname{tg} \varphi_0} \right), \tag{3.1}$$

where $\chi(\tau) = 2\tau - \tau^2/2$ in the band $0 \le \tau \le 2$, while for $\tau \gg 1$ (actually for $\tau > 2$) the following asymptotic formula holds

$$\chi(\tau) = \frac{\pi}{2} \left\{ 1 + 2 \operatorname{Real} \left[\frac{A}{B} e^{B\tau} \right] + \frac{1}{4\tau^2} - \frac{3}{4\tau^4} \left[\ln 4\tau - \frac{29}{24} \right] \right\},$$

where $A = 0.76\ e^{-0.723i}$ and $B = 1.306 e^{2.12i}$ are complex numbers. Substituting (3.1) into (2.5) and (2.6), we obtain

$$L(x, t) = 4\alpha \operatorname{tg} \varphi_0 N(x, t) H(t - k_1 x), \quad c_y(t) = \int_0^{x_1} N(x, t)\, dx,$$

$$m_z(t) = -\int_0^{x_1} x N(x, t)\, dx,$$

$$N(x, t) = 2x\chi \left(\frac{t - k_1 x}{x \operatorname{tg} \varphi_0} \right) + \frac{x - \mathrm{M}t}{\mathrm{M} \operatorname{tg} \varphi_0} \chi' \left(\frac{t - k_1 x}{x \operatorname{tg} \varphi_0} \right),$$

$$x_1 = \min (t/k_1, 1).$$

As t grows the coefficients c_y and m_z asymptotically approach their stationary values $c_{y\infty} = 2\pi\alpha \tan \varphi_0 = (1/2)\pi\alpha\lambda$, $m_{z\infty} = -(1/3)\pi\alpha\lambda$ ($\lambda = 4 \tan \varphi_0$ is the wing span).

Graphs displaying the change in the nonstationary aerodynamic characteristics C(t) and m(t) with time and their comparison with the quasistationary approximation are given in Fig. 4 ($M = 1.5$, $\gamma = 20°$, curves 1-3 correspond to 3, 6, 10°). It is seen that the nonstationarity of the flow influences a small span wing much more strongly than a wing with supersonic leading edges. It results in substantially nonmonotonic dependence of the aerodynamic coefficients on the time with quite definite "discards" relative to the steady values. The maximal values of C(t) and m(t) are achieved at the time of total enclosure of the wing by the shock. As parametric computations showed, the relative magnitudes of the "discards" (the differences $C_{max} - C_\infty$, $|m|_{max} - |m|_\infty$) grow as the angle at the wing apex diminishes (Fig. 4) as do also the Mach flight number and the angle of shock incidence.

It is seen from the comparison executed in Fig. 4 that the quasistationary approximation yelding the monotonic change in the aerodynamic characteristics does not permit determination of their maximal values and the results of a computation of the substantially nonstationary flow must be used for this.

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